Technical Comments

Comment on "Stability of Multidimensional Linear Time-Varying Systems"

Peter C. Müller*
University of Wuppertal
Wuppertal, Federal Republic of Germany

IN the paper[1] by Shrivastava and Pradeep[1] seven theorems on stability of multidimensional linear second-order systems with time-varying coefficients have been discussed. Unfortunately, all statements on asymptotic stability are useless because they are based on contradictory conditions. For example, theorem 1 is considered where, within the "sufficient" conditions for asymptotic stability, a positive definite mass matrix and a negative definite time derivative of the mass matrix are simultaneously required:

$$M(t)>0, \qquad M'(t)<0 \tag{1}$$

With respect to the definitions of positive or negative time-variant functions,[2] the requirements (1) imply the existence of constant positive definite matrices M_1 and M_2 such that

$$M(t)\geq M_1>0, \qquad M'(t)\leq -M_2<0 \tag{2}$$

for all $t\geq t_0$. Therefore we have

$$M(t)=M(t_0)+\int_{t_0}^{t} M'(\tau)\mathrm{d}\tau\leq M(t_0)-M_2(t-t_0) \tag{3}$$

At least for

$$t>t_0+\frac{\lambda_{\max}[M(t_0)]}{\lambda_{\min}(M_2)} \tag{4}$$

inequality (3) leads to a contradiction of $M(t)>0$.

Such contradictory conditions [Eq. (1)] also appear in theorems 1-5 and corollaries 1.1 and 3.1. Therefore, all results on asymptotic stability cannot be used.

If the asymptotic stability of mechanical systems has to be shown by the suggested Liapunov functions V, then the results of Krasovskii[2] have to be applied, showing that each nontrivial half-trajectory does not satisfy identically $\dot{V}\equiv 0$. In the case of linear systems, this analysis can be replaced by certain observability or controllability conditions. For time-invariant mechanical systems particularly, this discussion leads to the notion of pervasive damping ensuring asymptotic stability.[3]

A last comment corresponds to the stability part of theorems 4 and 5. The well-known lemma leads to the very conservative conditions (25) and (27). For example, condition (27) requires the existence of a matrix L with

$$A=K'L \tag{5}$$

i.e., the condition

$$\operatorname{rank} K'=\operatorname{rank}[K'A] \tag{6}$$

is necessary for the possibility that Eq. (27) may hold. In application, this rank condition is very rarely satisfied for systems with many degrees of freedom. In particular, the theorems cannot be applied to time-invariant circulatory systems. Further results on this subject are reported by the author.[3]

Received Feb. 19, 1986.
*Professor, Department of Safety Control Engineering.

References

[1]Shrivastava, S.K. and Pradeep, S., "Stability of Multidimensional Linear Time-Varying Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 8, Sept.-Oct. 1985, pp. 579-583.
[2]Krasovskii, N.N., *Stability of Motion,* Stanford University Press, Stanford, CA, 1963, pp. 8, 66-68.
[3]Müller, P.C., *Stabilität und Matrizen,* Springer-Verlag, Berlin, 1977.

Reply by Authors to P.C. Müller

Shashi K. Shrivastava* and S. Pradeep†
Indian Institute of Science, Bangalore, India

WE wish to thank Professor Müller for his interest in our paper.[1] However, we do not agree with his comment that "M' being negative definite is inconsistent with M being positive definite." Here is a counterexample:

$$M=\begin{bmatrix} 4e^{-4t} & 0 \\ 0 & 2e^{-2t} \end{bmatrix}>0$$
$$M'=\begin{bmatrix} -16e^{-4t} & 0 \\ 0 & -4e^{-2t} \end{bmatrix}<0 \qquad \text{for all } t\in[0,\infty)$$

One can find many such examples. Therefore the "sufficient" conditions for asymptotic stability in theorems 1-5 and corollaries 1.1 and 3.1 are applicable in a number of situations.

The results of Krasovskii[2] cannot be applied to the system under consideration. The analysis in Ref. 2 is restricted to equations with periodic/constant coefficients, for which a number of well-known theorems exist, whereas the theorems of Ref. 1 deal with arbitrarily time-varying systems.

With regard to the last comment, the authors fail to understand the significance of condition (5) of Müller. Equation (27) of Ref. 1 is

$$\left[\begin{array}{c|c} -K' & -A \\ \hline A & 2D-M' \end{array}\right]\geq 0 \qquad \text{for all } t\in[0,\infty) \tag{3}$$

It is not clear why this requires the existence of a matrix L with

$$A=K'L \tag{2}$$

If K' is nonsingular (as is the case in most time-varying systems), then, trivially, L exists and $L=(K')^{-1}A$ (although it is not clear what the role of L is). If K is singular, then L does not exist. However, this does not prevent Eq. (1) from

Received May 6, 1986.
*Professor. Associate Fellow AIAA.
†J.R.D. Tata Research Fellow.